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TWO-PHASE FLOWS WITH FRICTION

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Results are presented of a study of the equations of one-dimensional steady two-phase flows, taking account of friction with the channel walls.

§1. One-dimensional steady flows of a wet vapor in thermodynamic equilibrium are studied. The thermal conductivity of the vapor, the volume of the liquid phase, and the difference between the phase velocities are not taken into account.

It is assumed that friction is the only uncompensated external action on the flow. These flows belong to the class of flows with one internal degree of freedom [1] — the phase transition — and one external action — friction.

The effect of friction appears to one degree or another in all motions of two-phase media in channels. The pressure drop in a channel due to the performance of work against friction is an important engineering characteristic. Many empirical relations are known for calculating the pressure loss due to friction [2]. However, each of these has a limited range of application and does not reflect the dynamics of the flow of a wet vapor. It is of interest to study the appropriate differential equations to determine the general qualitative character of flows of a wet vapor acted upon by frictional forces following any resistance laws for all possible values of the parameters of a two-phase medium compatible with the conditions of the problem posed.

The results of the analysis can be applied to the little studied but practically important theoretical problem of the efflux of a self-evaporating liquid. This flow is, on the whole, nonequilibrium, but for a sufficiently long channel it has a quasiequilibrium boundary region of wet vapor [3]. The cross-sectional area of the channel occupied by the wet vapor and also its mass flow rate vary from section to section as a result of the vaporization of the metastable liquid at the center of the channel. The temperature of the liquid remains practically constant [3], and it will be shown later that this leads to the compensation of the geometric action of the emerging stream on the flow of wet vapor in the boundary region. The equations describing this flow are the same as the equations of equilibrium two-phase flow with friction.

§2. The equations of continuity, motion, and energy corresponding to the equilibrium flow of a wet vapor in a channel of constant cross section have the form

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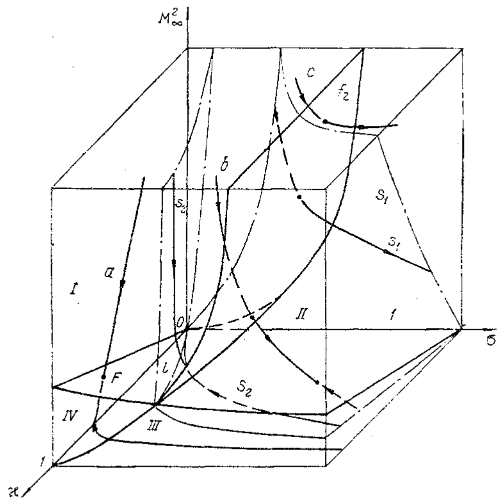

Fig. 1. Phase diagram of flow.

$$\frac{d\rho}{\rho dx}+\frac{du}{udx}=0, \tag{1}$$

$$\frac{dP}{dx}+\rho u\frac{du}{dx}+\rho_1\frac{dL}{dx}=0, \tag{2}$$

$$c_p\frac{dT}{dx}+u\frac{du}{dx}+r\frac{d\varkappa}{dx}=0. \tag{3}$$

The equation of continuity for the boundary region of a wet vapor formed in the outflow of a self-evaporating liquid has the form

$$\frac{d\rho}{\rho dx}+\frac{du}{udx}+\frac{dA}{Adx}=\frac{dG}{Gdx}. \tag{3a}$$

The total mass flow rate through a channel of cross-sectional area S (assumed constant) is

$$G_0=A\rho u+(S-A)\rho_2 u_2.$$

If it is assumed that the heat flux resulting from the temperature drop between the liquid and the wet vapor is all expended in evaporating the metastable liquid, the velocity of the liquid is constant, since the stagnation enthalpy and the temperature of the liquid are not changed. Therefore, the total flow rate of the mixture is $G_0 = S\rho_2 u_2$; i.e., $\rho u = \rho_2 u_2$. In addition, $u_2\rho_2\, dA/dx = dG/dx$, and, consequently, $(dA/dx) = dG/dx$,

Thus Eq. (3a) is the same as (3). The equations of motion and energy (2) and (3) also retain their form for the flow of wet vapor in the efflux of a self-evaporating liquid. The system (1)-(3) together with the Clapeyron—Clausius equation

$$\frac{dP}{dT}=\frac{r\rho_1}{T} \tag{4}$$

and the equation of state of an ideal gas

$$\frac{dP}{Pdx}+\frac{d\rho_1}{\rho_1 dx}=\frac{dT}{Tdx} \tag{5}$$

can be solved for the derivatives

$$\frac{d\xi_i}{dx}=\xi_i\frac{F_i}{F}\frac{\gamma}{a_\infty^2}\frac{dL}{dx},\ i=1-6, \tag{6}$$

where

$$\xi_1=u_1;\ \xi_2=\rho;\ \xi_3=P;\ \xi_4=\varkappa;\ \xi_5=T;\ \xi_6=M_\infty^2;\ F_1=-\varkappa f_1;\ F_2=-F_1;$$

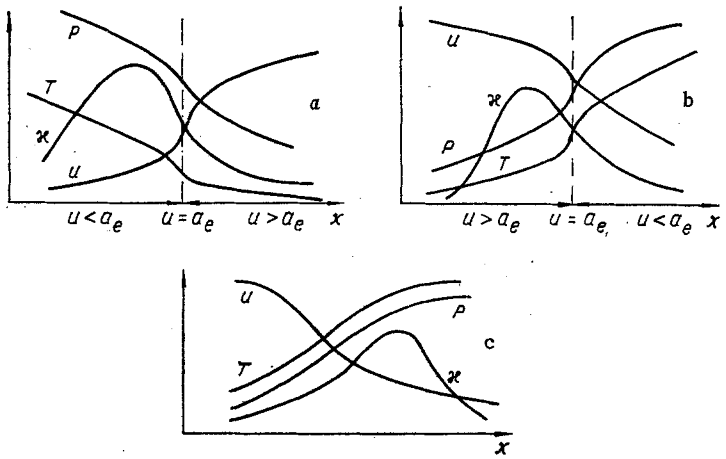

Fig. 2. Integral curves corresponding to various types of flow.

$$F_3 = \frac{\gamma\varkappa}{\gamma-1} f_3;\ F_4 = \sigma\varkappa f_2;\ F_5 = \sigma\varkappa f_3;\ F_6 = -\varkappa\sigma f_3 - 2\varkappa f_1;\ F = \gamma M_\infty^2 f_1 - \frac{\gamma\varkappa}{\gamma-1} f_3;$$

$$f_1 = \sigma^2 + \varkappa\left(\frac{\gamma}{\gamma-1} - \sigma\right);\ f_2 = (\gamma-1) M_\infty^2\left(\frac{\gamma}{\gamma-1} - \sigma\right) - \sigma;\ f_3 = (\gamma-1)\sigma M_\infty^2 + \varkappa;\ \sigma = \frac{Tc_p}{r}$$

is the dimensionless temperature of the medium; $a_\infty = \sqrt{\gamma RT}$ is the sound speed in a two-phase medium for an infinite frequency of the sound vibrations [4]; and $M_\infty = u/a_\infty$.

If we introduce a new independent reduced length variable oriented along the channel

$$\chi = \int_0^L \frac{\gamma}{a_\infty^2}\, dL, \tag{7}$$

the last three equations of system (6) form an independent system of ordinary differential equations [5]:

$$\frac{d\varkappa}{d\chi} = \varkappa\frac{F_4(\varkappa, \sigma, M_\infty^2)}{F(\varkappa, \sigma, M_\infty^2)},\quad \frac{d\sigma}{d\chi} = \sigma\frac{F_5(\varkappa, \sigma, M_\infty^2)}{F(\varkappa, \sigma, M_\infty^2)},$$
$$\frac{dM_\infty^2}{d\chi} = M_\infty^2\frac{F_6(\varkappa, \sigma, M_\infty^2)}{F(\varkappa, \sigma, M_\infty^2)}. \tag{8}$$

§3. Analysis of the solutions of system (8), as in [6] which is devoted to the study of nonequilibrium two-phase flow, is performed in three-dimensional phase space with the coordinates M_∞^2, σ, and $\varkappa$. The trajectories of this space represent the solutions of systems (8) and (6).

We determine the "zero" surfaces on which the derivatives of Eqs. (8) and (6) vanish. These surfaces are the following: the surface $f_2 = 0$, which passes through the axis $M_\infty^2 = 0$, $\sigma = 0$, asymptotically approaching the plane $\sigma = 1$ (Fig. 1); on this surface $d\varkappa/d\chi = 0$; the plane $\sigma = 0$ on which $d\sigma/d\chi = d\varkappa/d\chi = 0$; the plane $M_\infty^2 = 0$ on which $dM_\infty^2/d\chi = 0$; the plane $\varkappa = 0$ on which $dM_\infty^2/d\chi = d\sigma/d\chi = d\varkappa/d\chi = 0$. This plane is the set of equilibrium positions of the system, stable for $M_\infty^2 < \sigma/[\gamma - \sigma(\gamma-1)]$, and unstable for $M_\infty^2 > \sigma/[\gamma - \sigma(\gamma-1)]$. The remaining "zero" surfaces lie outside the positive octant of phase space bounded by the plane $\varkappa = 1$.

Since the equilibrium positions of system (8) form a plane in three-dimensional phase space, the trajectory is a straight line [7] in the neighborhood of each equilibrium position. Along these trajectories the point representing the state of the system approaches the equilibrium position asymptotically or moves away from it.

Points on the surface $F(\varkappa, \sigma, M_\infty^2) = 0$ are singular; at these points the derivatives of system (8) and (6) have two-sided infinite discontinuities. This corresponds to the "crisis" phenomenon of steady flow — the existence of limiting states at the channel exit where the flow velocity becomes equal to the local sound velocity

[8]. Within the framework of the problem posed on equilibrium two-phase flow the frequency of vibrations propagating in the stream must be rather small. In this case the sound speed in wet vapor must be calculated for zero frequency of the sound vibrations. Such an equilibrium sound speed is calculated in [9, 10, 4]:

$$a_e = \frac{\varkappa (RT)^{1/2}}{\left[\left(1 - 2\sigma \frac{\gamma - 1}{\gamma}\right)\varkappa + \sigma^2 \frac{\gamma - 1}{\gamma}\right]^{1/2}}. \tag{9}$$

Substituting Eq. (9) into the equation $F(\varkappa, \sigma, M_\infty^2) = 0$ actually shows that the flow velocity u in the section where $F = 0$ is equal to the equilibrium sound velocity a_e.

The surfaces $f_2 = 0$ and $F = 0$, which intersect along the line l, divide the phase space into four regions, in each of which the directions of change of the phase coordinates along the channel are specified in Fig. 1 by arrows at the surfaces. Regions I and III correspond to flows with evaporation of drops of liquid in the wet vapor and regions II and IV, to flows with condensation of vapor on drops for supersonic and subsonic velocities, respectively.

It is easy to see from Fig. 1 that there are three types of qualitatively different phase-space trajectories (a, b, c). These three types of trajectories are separated by limiting curves s_1 and s_2 forming limiting surfaces S_1 and S_2. For a sufficiently long channel trajectories a and b bring both supersonic and subsonic flows to "crisis." Supersonic flows along trajectories a and subsonic flows along trajectories b are accompanied by evaporation of drops of liquid. In subsonic flows along trajectories a and supersonic flows along trajectories b the flows produce condensation of vapor in the approach to "crisis." Trajectories c correspond to supersonic flows from positions of unstable equilibrium of the system to positions of stable equilibrium. If a discontinuous transition does not occur in subsonic flows, a flow "crisis" does not arise for channels of any length.

The limiting curves s_2 are different in that when they reach the singular surface $F = 0$ the derivative $d\varkappa/d\chi$ has a one-sided discontinuity. The limiting curve s_1 does not reach the surface $F = 0$, but goes to infinity along the coordinate σ. Parts of the phase-space trajectories in the vicinity of the plane $\varkappa = 0$ have no physical meaning, since two-phase flows with a vapor content close to zero do not have a disperse structure and are not described by Eqs. (1)-(3).

Graphs of the variation of the flow parameters along the characteristic trajectories a, b, c are shown in Fig. 2. It is clear from these curves that in supersonic flows the velocity decreases, and the pressure, temperature, and density of the medium are increased; in subsonic flows the velocity increases, and the pressure, temperature, and density decrease.

§4. Study showed that independently of the laws of resistance for the flow of an equilibrium disperse two-phase stream there are only five different types of supersonic flow (trajectories a, b, c, s_1, s_2) and three types of subsonic flows (trajectories a, b, s_2). The character of the variation of the parameters of the medium — velocity, pressure, density, and temperature — for all types of flow are the same as in uniform flows with friction. The essential difference between the two-phase flows considered and one-phase flows is the possibility of supersonic flow without crisis (trajectory c). This property becomes important in the study of streams of a self-evaporating liquid. The effect on the flow of a two-phase medium with one internal degree of freedom — phase transition — leads to evaporation of the liquid in the vapor in supersonic flows for velocities $u > T\left(\frac{c_p}{\frac{r}{c_p}\frac{\gamma}{\gamma-1} - T}\right)^{1/2}$ and in subsonic flows for velocities $u < T\left(\frac{c_p}{\frac{r}{c_p}\frac{\gamma}{\gamma-1} - T}\right)^{1/2}$. If these conditions are not satisfied, vapor condenses. The rule mentioned holds also for flows of wet vapor formed in a stream of self-evaporating liquid.

NOTATION

ρ, u, T, density, velocity, and temperature of medium; ρ_1, P, vapor density and pressure; r, c_p, heat of phase transition and specific heat of vapor at constant pressure; $\varkappa$, L, vapor content at outlet and work of stream against friction; x, independent variable oriented along channel; A, G, cross-sectional area of channel occupied by wet vapor and its flow rate; ρ_2, u_2, density and velocity of metastable liquid; S, cross-sectional area of channel; γ, adiabatic index of dry vapor; R, specific gas constant; a_e, equilibrium speed of sound.

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ASYMMETRY OF THERMOGRAVITATIONAL CONVECTION

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The nature and intensity of convective motion in a rectangular region with moving boundaries of the solidification front are studied by the finite-difference method.

Thermal convection in regions with moving boundaries of the solidification front, its nature, and intensity have an important effect on heat and mass transfer in the liquid phase, on the redistribution of an admixture in the solid crust, and on the macrostructure of the finished casting. The three-dimensional problem of unsteady thermal convection in a rectangular prism was formulated and solved in [1]. The plane case of thermal convection was analyzed in [2] and that for a cylindrical region, in [3]. The considerable divergence in the calculated results and in some cases the contradiction of the conclusions indicate the necessity of further study of this problem with the aim of clarifying the determining factors of the process of thermal convection.

A region of rectangular cross section, semiinfinite along the coordinate η_2, was chosen for study in the present report. The region is filled with a stationary homogeneous melt with an initial temperature T_0 higher than its crystallization temperature.

Proceeding from the assumption that the vertical axis $0\eta_3$ is the axis of symmetry of the convective streams, one of the halves of the region under consideration is represented in Fig. 1. The dimensions in the diagram are relative, with the horizontal width being taken as the characteristic size, so that $l_1 = 1$.

At a time $\tau > 0$ the temperature of the boundaries of the region is abruptly reduced to the crystallization temperature, as a result of which the solid phase is formed at the periphery. The solidification front is assumed to be plane. The dimensions of the liquid phase along the coordinates η_1 and η_3 and the thickness of the top crust are assumed to be known functions of time:

$$\varepsilon_1 = 1 - k_1\sqrt{\mathrm{Fo}};\ \varepsilon_3 = l_3 - k_2\sqrt{\mathrm{Fo}},\ H = k_3\sqrt{\mathrm{Fo}}, \tag{1}$$

where k_1, k_2, and k_3 are solidification coefficients.

In the Boussinesq approximation the initial system of equations in dimensionless vector form is written

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